Few-Body Syst
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Modern Chiral Forces Applied to the Nucleon–Deuteron Radiative Capture

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Abstract The chiral nucleon–nucleon interaction with semi-local regularization up to the fifth order of chiral expansion is applied to the nucleon–deuteron radiative capture process. Our theoretical approach is based on the formalism of Faddeev equations and the Siegert theorem is exploited to construct the electromagnetic current operator. The very weak dependence of the differential cross section on values of the regularization parameter is observed. This suggests that the improved chiral two-body interaction is a promising starting point to study electromagnetic processes at low energies.

1 Introduction

The recent progress in constructing chiral nuclear interactions resulted in the new version of the nucleon–nucleon (NN) potential [1,2]. This interaction is an improved version of the force derived in earlier works, starting from [3,4], see [5] for extensive review of this approach. The new model presented in [1,2] includes all terms of the chiral expansion up to N^{4}LO and benefits from the improved way in which values of the low energy constants in the long range part of the interaction are established. Namely, in Ref. [1,2] the values of these constants are taken directly from the pion-nucleon scattering without additional fine tuning applied in the previous model. Another difference compared to the older approach is the regularization method used. In the

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previous model a nonlocal regularization in momentum space has been applied. This means, that the matrix elements of the potential V: $\langle \mathbf{p}|V|\mathbf{p'}\rangle$ have been multiplied by an exponential factor $\exp(-(p^6 + p'^6)/\Lambda^6)$ where $\mathbf{p'}$ and $\mathbf{p}$ are the relative momenta of nucleons in the initial and final state, respectively, $p' = |\mathbf{p'}|$, $p = |\mathbf{p}|$ and $\Lambda \approx 550$ MeV is the regularization parameter. Such a regularization, implemented in the same way for all partial waves, leads to unwanted artifacts in the long-range part of the NN force and does not completely eliminate the unwanted short-range components of the two-pion exchange. Further, the corresponding nonlocal regularization has been used for the three-nucleon (3N) force [6], what influenced the description of observables in the 3N sector, see [7], and [8] for applications in the deuteron breakup process and [9–11] for the electromagnetic processes including proton–deuteron radiative capture as well as ^{2}H and ^{3}He photodisintegration reactions. These works have revealed that the cut-off dependence of the studied there model of nuclear forces is too strong (especially at N^{3}LO) and precludes precise conclusions about investigated processes.

In the recent model of Refs. [1,2] another regularization method is used. Namely, the long-range part of the interaction in coordinate space is multiplied with the function $f(r) = (1 - \exp(-r^2/R^2))^6$, while contact interactions are regularized using a non-local Gaussian regulator in momentum space. The values of the cutoff parameter R are chosen in the range of 0.8–1.2 fm, however they do not describe the two-nucleon phase shifts equally well - the best description (up to $E_{\mathrm{lab}} = 300$ MeV) is obtained for R of 0.9 and 1.0 fm.

The precise description of 3N systems is more challenging. Even restricting theoretical calculations to the two-body phenomenological forces only, due to the important role of the off-the-energy-shell parts of the interaction, one observes clear model dependence. It was illustrated many times, for example in [12,13], clearly pointing to the necessity of improvement of phenomenological forces employed in these calculations. The chiral interaction has more solid theoretical foundations and in principle should approach the exact theory. However, regulator functions, unavoidable in practical calculations, introduce unwanted uncertainties. While in nucleon–deuteron (Nd) elastic scattering this uncertainty is on the tolerable level for both regularization schemes [6,14], in the Nd radiative capture as well as in the ^{2}H and ^{3}He photodisintegration processes the cut-off dependence of predictions based on the nonlocal regularization is huge, even at low energies [9–11].

In this paper we test the regulator dependence of predictions based on the local regularization for the radiative Nd capture process. We briefly summarize our method in Sec. 2 and present results, at two reaction energies, for the differential cross section in Sec. 3. Other observables and more detailed discussion comprising also photodisintegration and weak muon-capture processes can be found in [15].

2 Formalism

Since our approach has been described in detail for example in [16,17], here we only remind the reader of our key equations for the two-body photodisintegration nuclear matrix element N^{μ}_{Nd}. If the 3N force is neglected, we simply get:

$$N^{\mu}_{Nd} = \langle \phi_{\mathrm{Nd}}|(1+P)\, j^{\mu}_{3N}|\Psi^{3N}_{\mathrm{bound}}\rangle + \langle \phi_{\mathrm{Nd}}|P|U^{\mu}\rangle\,, \quad (1)$$

$$|\, U^{\mu}\,\rangle = tG_0\,(1+P)\, j^{\mu}_{3N}\,|\,\Psi^{3N}_{\mathrm{bound}}\,\rangle + tG_0 P\,|\,U^{\mu}\,\rangle\,, \quad (2)$$

where j^{μ}_{3N}, P, t and G_0 are the electromagnetic current operator, the permutation operator, the t-matrix operator (built from the two-body interaction) and the free 3N propagator, respectively. Further, $|\Psi^{3N}_{\mathrm{bound}}\rangle$ and $|\phi_{\mathrm{Nd}}\rangle$ denotes the initial 3N bound state and the final Nd state, respectively. The nuclear matrix element for the radiative Nd capture can be obtained from N^{μ}_{Nd} by time-reversal symmetry.

The electromagnetic current operator consistent with the strong interaction with the semi-local regularization has not been derived yet (see [18,19] for the first steps in this direction). Thus we follow [16,17] and use the standard single nucleon current operator together with the Siegert theorem to include implicitly many-body contributions to the electromagnetic current.

3 Results and Summary

In Fig. 1 we exemplify the unsatisfactory description of the Nd radiative capture cross section delivered by the nonlocally regularized chiral forces [4]. The cyan (red) band comprises N^{2}LO (N^{3}LO) predictions obtained with various values of the cut-off parameters, as suggested in [4]. It is clear that the width of the bands at both orders of the chiral expansion and for both processes presented here (the neutron-deuteron capture at

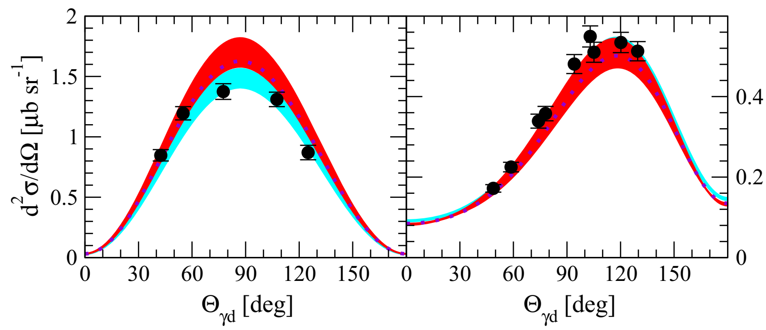

Fig. 1 The differential cross section $d^2\sigma/d\Omega$ for the n + d $\rightarrow^3$ H + γ reaction at $E_n = 9.0$ MeV (*left*) and for the p + d $\rightarrow$ ^{3}He + γ reaction at $E_d = 95$ MeV (*right*). Theoretical predictions have been obtained using the chiral forces with nonlocal regularization [4]. The *cyan* and *red* bands cover predictions at N^{2}LO and N^{3}LO obtained with different values of the regularization parameters, respectively. The *thick violet dotted curves* show predictions based on the AV18 potential. The *circles* represent data from Ref. [20] (*left*) and from Ref. [21] (*right*)

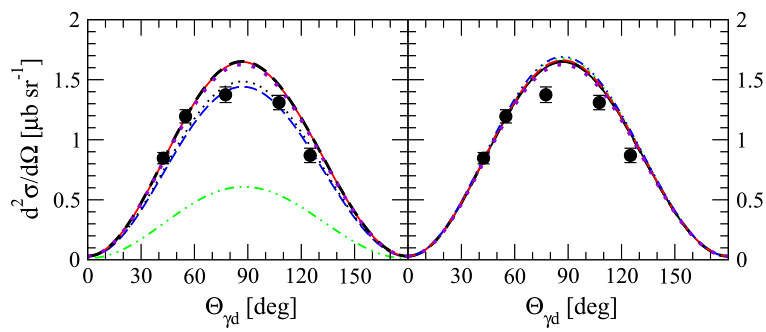

Fig. 2 The differential cross section $d^2\sigma/d\Omega$ for the n + d $\rightarrow^3$ H + γ reaction at $E_n = 9.0$ MeV. The *left column* shows the convergence of predictions at $R = 0.9$ fm with respect to the order of chiral expansion. The *double-dotted-dashed green*, *dashed blue*, *dotted black*, *solid red* and *thick dashed black curves* represent predictions at LO, NLO, N^{2}LO, N^{3}LO and N^{4}LO, respectively. The *right panel* shows the dependence of the predictions at N^{4}LO on the value of the R parameter. The *dotted green*, *thick dashed black*, *solid black*, *dashed red* and *double-dotted-dashed blue curves* correspond to predictions with $R = 0.8$, 0.9, 1.0, 1.1 and 1.2 fm, respectively. Note that the *thick dashed black curve* shown in both panels represents the same predictions. Also the thick *violet dotted curve* which represents the AV18 predictions appears in both panels and in the *left panel* of Fig. 1. The data are from Ref. [20]

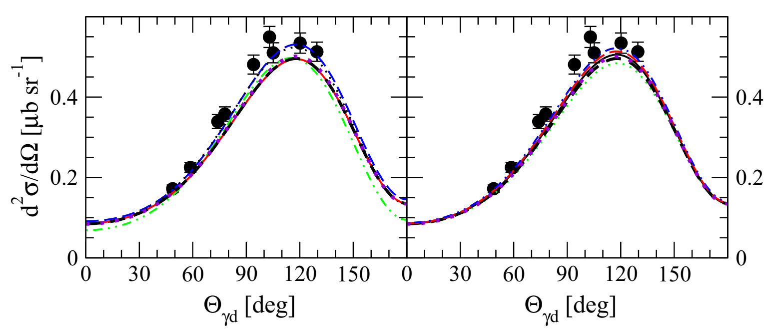

Fig. 3 The differential cross section $d^2\sigma/d\Omega$ for the p + d $\rightarrow^3$ He + γ reaction at $E_d = 95$ MeV. Curves as in Fig. 2; the AV18 based predictions are the same as in the right panel of Fig. 1. The data are from Ref. [21]

$E_n = 9$ MeV and the proton–deuteron capture at $E_d = 95$ MeV) is too big to allow one to investigate details of the underlying Hamiltonian.

A big improvement can be observed when the chiral force with the semi-local regularization is used. In Figs. 2 and 3 we show predictions for the same cross sections as in Fig. 1. Now, instead of bands showing the dependence on the value of the regularization parameter R, we present in the right panels separate lines for $R = 0.8$, 0.9, 1.0, 1.1 and 1.2 fm. Since the values $R = 0.9$ fm and $R = 1.0$ fm give the best description of the NN system, combining all curves into one band could be somewhat misleading. We observe that all values of R lead to nearly the same values of the cross sections. Moreover, at a lower energy results with extreme values $R = 0.8$ fm (green curve) and $R = 1.2$ fm (blue curve) lie slightly above the remaining, practically overlapping lines. Also at $E_d = 95$ MeV predictions based on $R = 0.8$ fm and $R = 1.2$ fm slightly deviate from the other ones, although the spread of predictions remains small.

In the left panels of Figs. 2 and 3 we show the convergence of predictions with respect to the order of chiral expansion. At both energies predictions obtained at N^3LO and N^4LO are practically indistinguishable. They differ from predictions at NLO and N^2LO, which are close to one another, which is expected due to the appearance of new contact interactions at N^3LO.

Results presented here and in Ref. [15] obtained with NN forces only confirm a good quality of the chiral force model proposed in Refs. [1,2]. However, the final judgment on quality and usefulness of semi-locally regularized interaction in analysis of electromagnetic three-nucleon reactions could be formulated only after inclusion of three-nucleon forces and consistent electromagnetic current to theoretical analysis. Such work is in progress.

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